Mathematical Learning Difficulties and PASS Cognitive Processes

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Abstract

This study examined the relationships between mathematical learning difficulties (MLD) and the *planning, attention, simultaneous, successive* (PASS) theory of cognitive processing. The *Cognitive Assessment System* (CAS) was used to measure the PASS processes for a group of 267 Dutch students with MLD who attended either general or special education. The results showed that students with MLD performed lower than their peers on all CAS scales and that the MLD group contained many students with cognitive weaknesses in planning or successive processing. Moreover, students who had specific difficulties with the acquisition of basic math facts, the automatization of such facts, or word-problem solving were found to have distinct PASS cognitive profiles. In order to investigate the relationships between cognitive abilities and improvement in the mastery of basic math facts and problem solving, 165 of the students with MLD were given a special multiplication intervention. It appeared that the effectiveness of this particular intervention did not differ across the groups of students with specific cognitive weaknesses.

Intelligence tests are mostly used to measure a student's general ability level. In the identification of learning disabilities (LD), IQ tests are also commonly used to compare a student's ability to his or her actual achievement. Unless the IQ–achievement discrepancy is beyond some predetermined value, a learning disability is not indicated (Mercer, 1997). The use of an IQ–achievement discrepancy, however, has been under attack for some time (e.g., Siegel, 1999; Stanovich, 1999). One reason is that the cutoff points for the general intelligence scores used to define LD are often based, at least in part, on tests that have a clear achievement component (Kaufman & Kaufman, 2001; Naglieri, 1999). Another reason is that intelligence cannot always be measured exactly; there is always some error, which complicates the use of an IQ score in, for example, an LD discrepancy formula. Conceptually, intelligence tests not only are used to measure the IQ–achievement discrepancy but also can be used to map children's cognitive strengths and weaknesses. Although findings generally do not support the use of IQ tests in this way (Kavale & Forness, 2000; Naglieri, 1999), recent research on cognitive processing has yielded promising results (Naglieri, 1999, 2000).

A growing body of research literature has described specific cognitive deficits in students with math learning difficulties (MLD). These students have been found to show deficits in working memory, storage and retrieval of math facts from long-term memory, number processing deficits, and problem-solving skills (e.g., Geary, Hamson, & Hoard, 2000; Ginsburg, 1997; Jordan & Hanich, 2000). In traditional tests of intelligence (e.g., the *Wechsler Intelligence Scale for Children,* third edition; WISC-III; Wechsler, 1991), these students are found to have low scores in Performance IQ relative to Verbal IQ, low scores on the perceptual organization factor, and relatively weak performance on digit span (Davis, Parr, & Lan, 1997; Geary et al., 2000; Jordan & Hanich, 2000). It should be noted, however, that these characteristics are based on population means. It is evident that the group of children with MLD is very heterogeneous. A distinction can be made between a group of students with math difficulties only and a group of students with both math and reading difficulties, with the latter group showing more general cognitive deficits, and the group with only math difficulties showing more specific cognitive deficits (Geary et al., 2000). The focus of this article is on students who have primarily MLD and on their specific cognitive characteristics.

The development of other approaches to intelligence testing, such as the *Kaufman Assessment Battery for Children* (K-ABC; Kaufman & Kaufman, 1983) and the *Cognitive Assessment System* (CAS; Naglieri & Das, 1997a), is of obvious relevance for both diagnostic (Naglieri, 1999) and instructional (Naglieri & Gottling, 1995, 1997; Naglieri & Johnson, 2000) purposes. Because these theory-based tests measure ability as a multidimensional concept, they may provide more information on specific components and processes than a test designed to measure general intelligence, such as the WISC-III. The specific information provided by these tests may be particularly useful during the diagnostic process, the design of instructional programs, and the development of specific interventions.

An example of such a new intelligence test is the CAS, which is based on a theory of cognitive processing that has redefined intelligence in terms of four basic psychological processes: *planning*, *attention*, and *simultaneous* and *successive* (PASS) cognitive processes. The CAS provides information on students' strengths and needs. Furthermore, CAS scores have been found to be strongly related to achievement ($r = .70$; Naglieri, 2001; Naglieri & Das, 1997b), which is quite remarkable, as the test does not contain the verbal and achievement components found in traditional measures of IQ (e.g., the WISC-III). This has led researchers in the Netherlands (Kroesbergen & Van Luit, 2002a; Kroesbergen, Van Luit, Van der Ben, Leuven, & Vermeer, 2000) to study the validity of the CAS when used in that country. This study examines the relationships between the CAS and MLD with particular intent to examine the potential of the PASS theory, on which the CAS is based, for the remediation of MLD.

The present investigation focused on a Dutch translation of the CAS, which consists of 12 subtests (3 subtests covering each of the four basic PASS processes). The subtests provide information on a child's cognitive functioning, which includes,

1. *planning* processes to provide cognitive control and use of processes and knowledge, intentionality, and self-regulation to achieve a desired goal;
2. *attentional* processes to provide focused, selective cognitive activity over time;

and two forms of operating on information, namely

3. *simultaneous* processes, by which the individual integrates separate stimuli into a single whole or group; and
4. *successive* processes, by which the individual integrates stimuli into a specific serial order that forms a chain-like progression (Naglieri & Das, 1997b).

Naglieri and Das (1997b) have found each of the four sets of PASS processes to correlate with specific types of achievement in math and other academic areas. Although all PASS processes are related to achievement, particular processes, such as planning, appear to be specifically related to particular aspects of academic performance, such as math calculation (Das, Naglieri, & Kirby, 1994). This specific example is theoretically logical because planning processes are required for making decisions about how to solve a math problem, monitor one's performance, recall and apply certain math facts, and evaluate one's answer (Naglieri & Das, 1997b). Simultaneous processes are particularly relevant for the solution of math problems, as they often consist of different, interrelated elements that must be integrated into a whole to attain the answer. Attention is important to selectively attend to the components of any academic task and focus on the relevant activities. Successive processes are also important for many academic tasks but, in mathematics, probably most important when the children do not follow the sequence of events and for the memorization of basic math facts. For example, when a child rehearses the math fact $8 + 7 = 15$, he or she learns the information as a serially arranged string of information that makes successive processing especially important. Successive processing is also important for the reading of words that are not known by sight and may, therefore, be particularly important for the solution of math word problems.

CAS scores have been found to correlate strongly with achievement scores (Naglieri & Das, 1997b). The overall correlation with the *Woodcock-Johnson Revised Tests of Achievement* (WJ-R; Woodcock & Johnson, 1989) skills cluster has been found to be .73. The correlations with mathematics skills have been found to range from .67 to .72, with the highest subscale correlations occurring for simultaneous processes and math (.62) and planning and math (.57). These correlations are quite high when compared to research with other intelligence tests (e.g., WISC-III, *Raven's Standard Progressive Matrices*). These findings suggest that the CAS is a good predictor of academic achievement in general and math achievement in particular.

Research has also suggested that a child's PASS profile is related to the effectiveness of particular intervention programs. Naglieri and Gottling (1995, 1997) and Naglieri and Johnson (2000), for example, have shown students to differentially benefit from instruction depending on their PASS cognitive profiles. The implication is that instruction can be made more effective when it is clearly matched to the cognitive characteristics of students. Along these lines, Naglieri and Johnson (2000) found the math computation of children with a planning weakness to benefit considerably from cognitive strategy instruction that emphasized planning; children with no planning weakness who nevertheless received the same planning-based instruction did not show the same level of improvement in math computation as the other children. Similar insights into the relations between the intelligence profiles of students and the effectiveness of particular intervention programs may aid the planning of remedial education programs and, therefore, call for further investigation.

In the present study, the relationships between PASS processes and mathematics achievement were investigated. Two questions were posed. The first question concerns the relation between cognition and MLD: Do students with MLD exhibit different PASS cognitive profiles than their typically achieving peers? To answer this question, a distinction was made between students who have difficulties learning basic math facts and students who have difficulties learning to solve math word problems. If these specific learning difficulties are associated with distinct cognitive profiles, then the CAS may also be of use for diagnostic purposes.

The second question concerns the relation between cognition and improvement in mathematics achievement. It is known that students with MLD are not very good at the automatization of math facts or word-problem solving (Jordan & Hanich, 2000; Naglieri & Johnson, 2000). In the present study, we examined whether students' cognitive profiles differentially related to the effectiveness of a particular math intervention focused on the promotion of both automaticity and adequate problem solving. The detection of such a difference would suggest that the CAS is useful not only as an instrument for the diagnosis of a student's cognitive characteristics, but also as a tool to effectively match the form of instruction or intervention to a student's particular needs.

Method

Participants

A total of 267 children with MLD participated in this study. The students in this group were selected on the basis of their low performance (below the 25th percentile) on a national criterion–based math test. Only students without serious reading or spelling difficulties were included. Because students with LD can be found in both general and special education elementary schools in the Netherlands, students from both types of schools were included. The group consisted of 137 students attending general education elementary schools (age M = 8.9, SD = 1.3; 44% boys, 56% girls) and 130 students attending special education elementary schools for students with learning or behavior problems (age M = 10.5, SD = 0.9; 73% boys, 27% girls). The CAS was also administered to a reference group, which consisted of 185 children without specific learning difficulties. These students were randomly selected within their schools. The mean age for this group of students was 9.8 years (SD = 1.2); 51% were boys, and 49% girls.

Procedure

The Dutch version of the CAS was administered by research assistants trained by the first author. The English version was adapted into a Dutch CAS following careful procedures by Kroesbergen and Van Luit (2002a). The preliminary reliability and validity analyses produced acceptable results. However, additional research with larger samples is still necessary to better evaluate the Dutch version of the CAS. It should be noted also that the study reported here is part of a larger research program concerned with the usefulness of the CAS in the Netherlands.

In order to address the second research question, part of the students with MLD were given special instruction focused on the learning of multiplication (n = 165; 86 general, 79 special education). These students were selected randomly from the group of 267 students with MLD. Attention was devoted to the automatized mastery of the basic multiplication facts and improvement of the students' use of multiplication strategies.

Intervention Program

For the intervention, the multiplication part of the *Mathematics Strategy Training for Educational Remediation* (MASTER) was used (Van Luit, Kaskens, & Van der Krol, 1993; see also Van Luit & Naglieri, 1999, and Kroesbergen & Van Luit, 2002b). This program was designed to encourage strategy use with multiplication problems. The program contains three series of lessons: (a) basic procedures; (b) multiplication tables; and (c) "easy" problems above 10×10. Each series teaches new steps for the solving of specific tasks. A series starts with an orientation phase, in which the child can solve the task with the help of materials. In the next phase, a connection is made to a mental solution. The subsequent control, shortening, automatization, and generalization phases are then completed.

The intervention involved 30 lessons of 30 minutes each, presented twice a week (4 months) to groups of five students. The emphasis in the lessons was on (a) the use of strategies, including metacognitive knowledge of how to select and apply the most appropriate strategies, and (b) automated mastery of the multiplication facts, as this knowledge is necessary for further learning and adequate problem solving. The discussion of possible solution strategies and procedures by the students was encouraged. The teacher assisted the students in such discussions, promoted reflection on the choices made, ensured that each student understood the different solutions, and prompted selection of the most efficient strategy. Students were thus taught to flexibly apply different strategies.

Measures

Cognitive Assessment System. The CAS is an individually administered test of ability for children ages 5 through 17 years and is organized into four scales (Planning, Attention, Simultaneous, and Successive) according to the PASS theory, each with a mean of 100 and SD of 15, and a Full Scale standard score. The test consists of 12 subtests; each subtest's scaled score is set at a mean of 10 and SD of 3. The CAS subtests are intended to be measures of the specific PASS process corresponding to the scale on which they are found rather than of specific abilities.

Planning scale. The Matching Numbers subtest consists of four pages, each with eight rows of numbers, six numbers per row. Children are instructed to underline the two numbers in each row that are the same. The numbers increase in length from one digit to seven digits. The subtest score is based on the combination of time and number correct for each page.

The Planned Codes subtest contains two items, each with distinct sets of codes and arrangements of rows and columns. A legend at the top of each page shows how letters correspond to simple codes (e.g., A, B, C, D correspond to OX, XX, OO, XO, respectively). Each page contains seven rows and eight columns of letters without codes. Children are permitted to complete each page in whatever fashion they desire. The subtest score is based on the combination of time and number correct for each page.

The Planned Connections subtest contains eight items. The first six items require children to connect numbers in sequential order. The last two items require children to connect both numbers and letters in sequential order, alternating between numbers and letters (for example, 1-A-2-B-3-C). The score is based on the total amount of time in seconds used to complete the items.

Attention scale. The Expressive Attention subtest uses two different sets of items, depending on the age of the child. Children age 8 years and older are presented with three pages. First, the child reads words such as *blue* and *yellow*; second, the child identifies the colors of a group of rectangles. Finally, the child identifies the color of ink in which certain words are printed. The performance on the last page is used as the measure of attention. The subtest score is based on the combination of time and number correct.

The Number Detection subtest consists of two pages of numbers that are printed in different formats. On each page, children are required to find a particular stimulus (e.g., the numbers 1, 2, and 3 printed in an open font) on a page containing many distractors (e.g., the same numbers printed in a different font). The score reflects the ratio of accuracy (total number correct minus the number of false detections) to total time for each item, summed across the items.

The Receptive Attention subtest is a two-page paper-and-pencil subtest. On the first page, letters that are physically the same (e.g., TT but not Tt) are targets, but on the second page, letters that have the same name (e.g., Aa but not Ba) are targets. The score reflects the ratio of accuracy (total number correct minus the number of false detections) to total time for each page, summed across pages.

Simultaneous scale. The Nonverbal Matrices subtest is a 33-item subtest that uses shapes and geometric designs that are interrelated through spatial or logical organization. Children are required to decode the relationships among the parts of the item and to choose the best of six options to occupy a missing space in the grid. The subtest score is based on the total number of items correctly answered.

The Verbal–Spatial Relations Subtest is composed of 27 items that require the comprehension of logical and grammatical descriptions of spatial relationships. Children are shown items containing six drawings and a printed question at the bottom of each page (e.g., "Which picture shows a circle to the left of a cross under a triangle above a square?"). The examiner reads the question aloud, and the child is required to select the option that matches the verbal description. The subtest score reflects the total number of items correctly answered within the 30-second time limit per item.

The Figure Memory subtest is a 27-item subtest. The child is shown a two-dimensional or three-dimensional geometric figure for 5 seconds. The figure is then removed, and the child is presented with a response page that contains the original design embedded in a larger, more complex geometric pattern. To be scored correct, all lines of the design have to be indicated without any additions or omissions. The score reflects the total number of items correctly identified.

Successive scale. The Word Series subtest requires the child to repeat a series of single-syllable, high-frequency words in the same order as stated by the examiner. Each series ranges in length from two to nine words, presented at the rate of one word per second. Items are scored as correct if the child reproduces the entire word series. The score is based on the total number of items correctly repeated.

The Sentence Repetition subtest requires the child to repeat 20 sentences that are read aloud. Each sentence is composed of color words (e.g., "The blue is yellowing"). The child is required to repeat each sentence exactly as it was presented. The subtest score reflects the total number of sentences correctly repeated.

The Sentence Questions subtest is a 21-item subtest that uses the same type of sentences as those in the Sentence Repetition subtest. Children from ages 8 to 17 are read a sentence and then asked a question about the sentence. For example, the examiner says, "The blue is yellowing," and asks the following question: "Who is yellowing?" The correct answer is "The blue." The subtest score reflects the total number of questions answered correctly.

Multiplication Tests. To measure the effects of the intervention, three multiplication tests were administered before and after the intervention period:

1. basic skills test, a test with 10 basic multiplications to measure knowledge of basic multiplication facts up to 10×10 (e.g., 5×3; 3×9);
2. automaticity test, a 2-minute speed test with 40 basic multiplication facts to measure the automatized knowledge of multiplication facts up to 10×10 (e.g., 6×4; 2×8); and
3. word problems test, a word-problem–solving test consisting of 20 relatively difficult multiplication problems (e.g., "The price of 2 glasses of juice is 3 euro; how much do you have to pay for 6 glasses?"; "Tina owns 43 comics, and Ann 52; John has twice as much comics as Tina; how many comics does John own?").

All three of these tests belong to the intervention program used in this study (Van Luit & Kroesbergen, 1999).

Results

In this study, the Dutch version of the CAS was used, although Dutch norms are not as yet available. For this reason, the experimental group was first compared to a Dutch reference group in addition to the U.S. norm. Before the research questions can be addressed, it must first be considered how the Dutch reference group performed in relation to the U.S. norms.

Performance of Reference Group

The Dutch version of the CAS was administered to a sample of Dutch children (n = 185) with no specific disabilities, and the scores for these children were compared to the U.S. norm. Table 1 shows the test scores for this reference group compared to the U.S.-based norms. Remarkably, the Full Scale score of 101.76 for the Dutch version of the CAS is very similar to the normative mean of 100 for the U.S. standardization sample. One-sample t tests nevertheless showed significant deviations from the U.S. norms for three of the four PASS scales and for the full scale. The difference for the Simultaneous scale was particularly large, with the mean score for the Dutch reference group being more than 5 points higher than the U.S. norm. It should be noted, however, that this difference represents only ⅓ *SD* for this scale. On the Successive scale, the Dutch children did not differ from the U.S. children. It is remarkable that the Planning scores for the Dutch reference group were below the U.S. average, whereas the scores of this group on the other scales were either average or above average. Table 2 shows the mean subtest standard scores and deviations from the normative value of 10. The means for 5 of the 12 subtests deviated significantly from 10, although the differences fell within 1 *SD* (3 points) from the mean.

PASS Cognitive Processes and MLD

In order to examine the relations between the various PASS cognitive processes and MLD, the CAS was administered to a group of students with MLD to determine if they showed PASS profiles different from those of their typically achieving peers (i.e., the reference group). In Table 3, the means and standard deviations for the different PASS scales are presented for the reference group and the group with MLD, with the latter group divided into students enrolled in special versus general education. Multivariate analyses of variance showed that the students with MLD performed significantly lower than their peers on all of the PASS scales, Full Scale $t(450) = 12.045$, $p < .001$. Further analyses showed similar differences on all 12 CAS subtests. Moreover, the students in special education showed lower scores than their peers with MLD in general education, Full Scale $t(265) = 8.074$, $p < .001$. Paired-samples t tests showed that in accordance with the results for the reference group, the scores of the group with MLD on the Simultaneous scale were relatively higher than their scores on the Planning, $t(266) = 10.293$, $p < .001$; Attention, $t(266) = 7.810$, $p < .001$; and Successive, $t(266) = 10.505$, $p < .001$, scales.

Given that students with MLD can be very diverse, we next distinguished different types of math difficulties:

1. students with difficulties in learning basic multiplication facts; these students scored at least 1 *SD* below the mean on the basic multiplication test but had received at least 1 year of multiplication instruction;
2. students with difficulties in reaching automatized mastery of basic facts; these students produced average scores on the basic multiplication test but scored at least 1 *SD* below the mean on the automaticity test; and
3. students with difficulties in learning to solve math word problems but no difficulties with basic multiplication facts; these students produced average scores on both the basic multiplication test and the automaticity test but scored at least 1 *SD* below the mean on the word-problem–solving test.

In the sample of 267 students with MLD, 45 students were found to clearly fit into one of these three groups (see Table 4). As can be seen from compar-

TABLE 1
CAS Standard Score and Differences from U.S. Normative Mean for Dutch Reference Group

Scale	*M*	*SD*	Range	Deviation from 100	*t*	*p*
Planning	98.27	11.00	63–129	–1.73	2.138	.034
Attention	101.96	11.69	71–138	+1.96	2.278	.024
Simultaneous	105.18	12.95	69–138	+5.18	5.445	.000
Successive	100.90	12.96	65–135	+0.90	0.941	.348
Full scale	101.76	11.54	72–137	+1.76	2.077	.039

Note. *N* = 185. CAS = *Cognitive Assessment System* (Kroesbergen & Van Luit, 2002a; Naglieri & Das, 1997a).

ing Table 3 and Table 4, no significant differences were found in the performance of the three groups on the four PASS scales relative to the total group of students with MLD ($p > .10$). Within-group analyses of variance showed that students with difficulties learning basic multiplication skills scored low on all four processes and had similar PASS profiles (no significant differences between the four processes; $p > .10$). Students with automaticity problems produced particularly low scores on the Planning, Attention, and Successive scales, together with relatively high scores on the Simultaneous scale: Planning–Simultaneous; $t(15) = 4.032$, $p = .001$; Attention–Simultaneous, $t(15) = 3.271$, $p = .005$; Successive–Simultaneous, $t(15) = 3.418$, $p = .004$. The group of students with difficulties solving word problems produced relatively lower scores on the Successive scale and relatively high scores on the Simultaneous scale, $t(13) = 2.701$, $p = .018$.

In the next set of analyses, we examined whether the group of students with MLD contained a greater number of students with cognitive weaknesses than the reference group. A cognitive weakness meant that the child's relevant scale score was significantly lower than the child's overall mean and less than 85 (1 *SD* below average). Inspection of Table 5 shows that more of the students in the group with MLD relative to the reference group had a cognitive weakness in planning, $\chi^2(2, N = 452) = 10.333$, $p = .006$, or in successive processing, $\chi^2(2, N = 452) = 33.936$, $p < .001$. The group of students with MLD in special education tended to have even more students with a successive processing weakness than the group of students with MLD in general education, $\chi^2(1, N = 267) = 7.119$, $p = .008$.

To summarize, students with different types of MLD produce lower PASS scale scores on average than their typically achieving peers and are also more likely to have a cognitive weakness in planning or successive processing. Similar results were found for both groups of students with MLD, although the deviations from the reference group were largest for the special education group.

Mathematics Performance and CAS

The second question to be addressed was whether a relation can be detected between improvement in mathematics performance (as a result of special instruction) and students' PASS scores. This was investigated by comparing the effects of the mathematics intervention on children with a specific cognitive weakness to its effects on children with no specific cognitive

TABLE 2
CAS Subtest Scaled Score Means, Standard Deviations, and *t*-test Results Compared to U.S. Norms

Scale/subtest	*M*	*SD*	*t*	*p*
Planning				
Matching Numbers	9.97	2.59	–0.170	.865
Planned Codes	9.05	1.89	–6.829	.000
Planned Connections	10.18	2.28	1.064	.289
Attention				
Expressive Attention	10.14	2.41	0.795	.428
Number Detection	10.85	2.32	4.977	.000
Receptive Attention	9.91	2.39	–0.492	.623
Simultaneous				
Nonverbal Matrices	10.89	2.69	4.484	.000
Verbal–Spatial Relations	9.99	2.72	–0.027	.978
Figure Memory	11.76	2.76	8.662	.000
Successive				
Word Series	9.78	2.46	–1.227	.221
Sentence Repetition	10.64	2.66	3.263	.001
Sentence Questions	10.17	2.62	0.870	.385

Note. CAS = *Cognitive Assessment System* (Kroesbergen & Van Luit, 2002a; Naglieri & Das, 1997a).

TABLE 3
CAS Standard Score Means and Standard Deviations for the Reference Group and the Group with MLD

Group	*n*	Planning *M*	Planning *SD*	Attention *M*	Attention *SD*	Successive *M*	Successive *SD*	Simultaneous *M*	Simultaneous *SD*	Full scale *M*	Full scale *SD*
Reference	185	98.3	11.0	102.0	11.7	100.9	13.0	105.2	13.0	101.8	11.5
MLD											
Total	267	89.0	12.2	91.1	12.6	87.9	13.9	97.9	12.1	88.3	12.0
Special education	137	85.6	11.8	86.6	12.9	82.6	12.7	95.0	12.2	82.9	11.2
General education	130	92.1	11.9	95.2	10.8	92.9	13.0	100.5	11.4	93.3	10.5

Note. CAS = *Cognitive Assessment System* (Kroesbergen & Van Luit, 2002a; Naglieri & Das, 1997a); MLD = math learning difficulties.

TABLE 4
CAS Standard Score Means and Standard Deviations for Subgroups of Students with Specific Math Difficulties

		Planning		Attention		Successive		Simultaneous		Full Scale	
Subgroup[a]	*n*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*	*M*	*SD*
Basic skills	15	90.7*	13.0	93.1	10.2	86.7*	16.1	93.1*	11.3	87.7*	12.8
Automaticity	16	84.7*	9.9	88.3	8.1	86.0*	10.5	98.3	9.2	85.3*	8.2
Word problems	14	96.1	14.2	92.6	11.7	93.4*	15.0	99.2	11.9	93.1*	13.4

Note. CAS = *Cognitive Assessment System* (Kroesbergen & Van Luit, 2002a; Naglieri & Das, 1997a). $N = 45$.
[a]As defined by low scores on respective math skills tests.

TABLE 5
Number and Percentage of Students in Different Groups with Specific Weaknesses on CAS Scales

		Planning		Attention		Successive		Simultaneous	
Group	*n*	*n*	%	*n*	%	*n*	%	*n*	%
Reference	185	9	4.9	4	2.2	12	6.5	6	3.2
MLD									
Total	267	38	14.2	17	6.4	65	24.3	8	3.0
Special education	130	18	13.8	10	7.7	41	31.5	4	3.1
Regular education	137	20	14.4	7	5.1	24	17.5	4	2.9

Note. CAS = *Cognitive Assessment System* (Kroesbergen & Van Luit, 2002a; Naglieri & Das, 1997a); MLD = math learning difficulties.

weakness. An overview of the students' scores on the three math achievement tests at pre- and posttest is presented in Table 6. As can be seen, the group of students with MLD improved as a result of intervention. However, no significant differences on students' improvement during intervention were found between the samples with a specific cognitive weakness: basic skills, $F(4, 161) = 0.158$, $p = .959$; automaticity, $F(4, 161) = 0.831$, $p = .507$; word problems, $F(4, 161) = 0.472$, $p = .756$. Likewise, no significant differences were found between the students enrolled in special versus general education: basic skills, $F(4, 161) = 0.011$, $p = .915$; automaticity, $F(4, 161) = 1.222$, $p = .271$; word problems, $F(4, 161) = 0.596$, $p = .441$.

Discussion

In this study, two main questions regarding the relation between mathematics learning difficulties (MLD) and cognition were investigated. The first question is relevant for the diagnostic procedure, whereas the second question concerned the effects of treatment.

First, we examined whether students with MLD exhibited cognitive profiles that are different from the cognitive profiles of their typically achieving peers. Students with MLD were indeed found to show relatively lower scores on the four PASS scales and, therefore, on the CAS Full Scale as well. The group of students with MLD performed highest on the Simultaneous scale, although the reference group also performed higher on this scale than the U.S. norm, which means that this result should be taken as tentative. Additional research and standardization of the CAS with respect to a Dutch norm is necessary to clearly settle this issue.

More detailed analyses of the present sample revealed a relation between specific math difficulties and specific PASS processes. It appeared that students with difficulties in learning the basic multiplication facts performed generally low on all four PASS processes, with no differences between the distinct processes. Contrary to the reference group, these students did not perform higher on simultaneous processing. Students with difficulties in the automatization of basic facts showed problems with successive processing, planning, and attention. These processes are particularly important for the automaticity test, because a time limit requires the efficient production of correct answers. Finally, students who had difficulties solving math word problems showed relatively weak attention and successive processing and relatively strong planning and simultaneous processing. Although both planning and simultaneous processing are important for the solution of math word problems, these findings suggest that attention and successive processing, which play an important role in reading, also play a key role in this type of math. The present results show that the PASS profiles of students with MLD differ from those of students with no such difficulties and, thus, demonstrate the potential diagnostic value of the CAS, especially in conjunction with other relevant information.

We also found that more of the students in the group with MLD had a cognitive weakness in planning (14%) or successive processing (24%). This is consistent with the results of a study conducted by Naglieri (2000). Planning is an important cognitive process in mathematics (Naglieri & Das, 1997b), along with simultaneous processing. In solving math word problems, successive processing also plays a critical role, which may explain the lower scores on this scale for the group of students with specific difficulties in solving math word problems. Given that a large part of the Dutch math curriculum consists of word problems, it is understandable that students with a successive processing weakness may encounter difficulties. However, the present results suggest that the group with MLD is heterogeneous, being composed of students with a specific planning weakness, students with specific successive processing weakness, students with generally low processing scores, and even a few students with attention or simultaneous processing weaknesses. The results also suggest that although a child's PASS profile alone is not sufficient to diagnose MLD, a child's PASS profile can help identify specific cognitive weaknesses and thereby facilitate both diagnosis and treatment.

In the second part of this study, we addressed the question of treatment. The relations between specific PASS cognitive profiles and the effectiveness of a special math intervention devoted to the learning of basic multiplication facts, the automatization of these facts, and word-problem–solving skills were carefully examined. Previous research showed that students with a cognitive planning weakness benefited from a cognitive intervention with specific attention to planning more than students without a cognitive weakness and more than students with other cognitive weaknesses (Naglieri & Johnson, 2000). However, these results were not confirmed in the present study. No differences in improvement were found between the different groups. An explanation for the discrepancy in the results of these different studies may lie in the fact that the intervention used in the present study was less focused on planning than the interventions used in previous research (e.g., Naglieri & Johnson, 2000). The intervention described here was mainly concerned with the acquisition of the basic math facts and the adequate use of strategies. Although planning is certainly part of strategy use, it was not explicitly taught. Furthermore, it seems of interest also to include other characteristics of students, such as their reading and spelling performance, to make a distinction between different groups. This will improve insight into the heterogeneity of the group of students with MLD and probably add knowledge about the specific needs of specific subtypes.

To conclude, the results of the present study revealed some important relationships between PASS cognitive processes and MLD. Although the relationships were not very strong, these findings nevertheless highlight the importance of particular cognitive processes for the functioning of students in certain areas of the mathematics curriculum. Attention and successive processing seem to be of importance for the solution of math word problems, for example, and planning appears to play a role in the automatization of basic facts in addition to attention and successive processing. Previous research has shown the CAS to be a valuable diagnostic instrument and also useful for the planning of special instruction or intervention (e.g., Naglieri & Johnson, 2000). We therefore encourage further research of PASS cognitive processes, together with other specific cognitive processes, such as working memory tasks. Future research might also address the specific difficulties that students encounter with the mathematics curriculum in connection with the development of special instructional methods based on PASS cognitive processing and its specific weaknesses.

TABLE 6
Pretest–Posttest Differences in Math Achievement Test Scores for Group with MLD

	Pretest		Posttest				
Test	*M*	*SD*	*M*	*SD*	*ES*	*t*	*p*
Basic skills	5.2	3.1	8.1	2.3	1.1	12.729**	< .001
Automaticity	16.7	7.6	26.2	7.9	1.2	16.931**	< .001
Word problems	7.8	5.3	11.7	5.3	0.7	12.595**	< .001

Note. MLD = math learning difficulties.
**$p < .001$.

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